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OPTIMAL ESTIMATION EQUATIONS FOR UNKNOWN BANDLIMITED SIGNALS*

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Abstract

The problem of approximating a bandlimited but otherwise arbitrary signal by a free solution to a linear, time-invariant, differential equation is solved. Optimal solutions for the equation parameters are derived for equations of any dynamic order. Applications are discussed and examples in state reconstruction and inverse filtering in the presence of unknown disturbances are given.

1. Introduction

In practice, one often has only partial information regarding the character of certain signals that are present in or act upon a system, examples being load torque disturbances in heavy machinery and electronic noise in certain semiconductor devices. For either signal processing or feedback control, it is important to be able to reconstruct such signals from output measurements alone. The question of this reconstruction is solved here for the class of such signals which are bandlimited, but otherwise unknown. The approach involves optimal approximation of the bandlimited signal, in the sense of integral-square bandpass error, as the homogeneous solution of a linear, time-invariant, ordinary differential equation with unknown initial conditions. The optimization determines the coefficients of the differential equation thus specifying the model.

The organization of the paper is as follows. In Section 2 further motivation is given for this work by showing how the theoretical problem arises from considerations in the application of observers and observer-controllers in feedback control.

The signal modeling problem is formulated in Section 3, where it is cast as an equivalent new filtering problem. In Section 4, a performance criterion and constraints are chosen, and the optimal solutions are found in closed form.

Two application examples are presented in Section 5, and conclusions are given in Section 6.

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2. Motivation: Observing the State of Systems with Unknown Inputs

Observer Design

When the state of a plant is not available to a control system for feedback, it may be estimated by a dynamic observer [1-3] or state estimator [4]. Providing that the plant is completely observable, an observer which monitors the plant inputs and outputs may be constructed to generate signals which converge arbitrarily rapidly to the system state, within the practical limitations of measurement noise and parameter errors.

When an observer estimate of the plant state is used for feedback in place of the state itself, the eigenvalues of the composite system are those of the observer (which may be chosen by the designer), together with those of the plant which would result if the state itself were fed back in place of the observed state.

In special cases, it is possible to observe the state of a system without having access to one or more of the system inputs [5-8]. Except in these cases, it is necessary to have all plant inputs available to the observer, and this requirement is a severe restriction on the usefulness of observers in many situations.

When all unknown inputs may be effectively characterized probabilistically, optimal stochastic filtering [9] is clearly indicated. In practice, however, there exist many situations, for example, structural systems with wind gust disturbances and chemical processes with reactant impurities, in which the system inputs are unknown (or poorly known) even in a statistical sense.

Observers Which Approximate Unknown Plant Inputs

A general method of accommodating unknown plant inputs in observers is to represent such signals as solutions of constant-coefficient, ordinary, linear, differential equations. The plant equations are augmented to represent the unknown inputs, and the resulting observer generates estimates of these inputs as well as the plant state.

This approach began with the work of Johnson [10-11], Pearson [12] and Davison [13-14], without explicit connection to observer theory.

Bryson and Luenberger took an observer viewpoint of a similar problem [15] and Young and Willems considered a more general problem class [16]. Hostetter and Meditch related Davison's work to the observer approach [17] and investigated the structure and properties of these observers in quite some detail [18-19].

Although there are situations where an inaccessible input signal is known to satisfy a specific differential equation, for example power line "hum", in most cases the differential equation for an unknown signal will be only an approximating equation, just as are the equations which model the plant. The question thus arises as to a "best" approximating equation for certain classes of input signals.

3. Problem Formulation

Representing Signals as Solutions to Differential Equations

Let an unknown signal f(t) be represented as a solution f'(t) to the scalar differential equation

$$a_n(d^nf'/dt^n) + a_{n-1}(d^{n-1}f'/dt^{n-1} + \dots + a_1(df'/dt) + a_of' = 0 \quad (1)$$

Ideally, the unknown signal f(t) would satisfy this differential equation exactly. But if a solution of the equation only approximates f(t), then

$$a_n(d^nf/dt^n) + a_{n-1}(d^{n-1}f/dt^{n-1}) + \dots + a_1(df/dt) + a_of = e(t), \quad (2)$$

where e(t) is an error signal, indicative of the quality of the approximation.

An Equivalent Filter

The relation (2) may be viewed as a filtering problem where f(t) is the filter input, e(t) is the filter output and the filter transfer function is

$$T(s) = a_ns^n + a_{n-1}s^{n-1} + \dots + a_1s + a_o.$$

One may then view the problem of determining parameters of an approximating differential equation as an equivalent problem of determining parameters of a filter, T(s).

This equivalent filter has all zeros and is thus not realizable as a finite-dimensional dynamic linear system [20]. But the equivalent filter is just an analytic convenience, not the end result.

Filter Characteristics for Bandlimited Signals

If the unknown signal f(t) is bandlimited, as most physical signals are [20], the filter T(s) should be chosen to have stopband characteristics over the range of frequencies present in f(t).*

*In linear systems of the integrating type (described by state equations without direct input-to-output coupling) the inputs, even if they are not bandlimited, may be considered to be bandlimited for all practical purposes as far as their effects upon the system state are concerned.

Were the filter transmittance zero over this band, f(t) would satisfy (2) exactly, with e(t)=0.

The thrust of classical filter theory over the years has been primarily toward practical, realizable designs such as all-pole filters and others in which the number of poles exceeds the number of zeros in the filter transfer function [21-22]. Well known techniques, however may be brought to bear upon the less conventional problem faced here.

4. Solutions for Optimal Filters

Performance Criterion and Constraints

Let the signal f(t) be bandlimited at radian frequency ω_o. Then a particularly useful and common measure of the performance of T(s) as a bandstop filter over the frequency range between $\omega = 0$ and $\omega = \omega_o$ is

$$J = \int_o^{\omega_o} |T(j\omega)|^2 d\omega.$$

Constraints are necessary, though, to yield useful solutions, and the particular restrictions $a_o = 0$ and $a_n = 1$ will be used. The transfer function will thus be restricted to be of the form

$$T(s) = s^n + a_{n-1}s^{n-1} + \dots + a_1s.$$

Requiring T(s) to have a zero at s = 0 reflects the desirability of zero transmittance of the filter for any constant component of f(t). This is to say that the approximating function to f(t), satisfying (1), includes a possible constant component. This requirement is particularly important in applications where signal offsets are likely and steady-state performance is of concern.

Fixing the coefficient of the highest power of s in T(s) is a simple means of avoiding the trivial solution

$$a_n = a_{n-1} = \dots = a_1 = a_o = 0$$

and is justified by the following observation: Since any equation (1) may be chosen in such a way as to include all solutions of a lower order equation, the equation of higher order gives at least as good an approximation to f(t) as does the equation of lower order.

Low Order Results

The optimum first order filter is constrained to have transfer function

$$T_1(s) = s,$$

which corresponds to the approximating differential equation

$$(df'/dt) = 0,$$

and an arbitrary constant approximating function.

The optimum second order filter is found to have transfer function

$$T_2(s) = s^2,$$

corresponding to

$$(d^2f'/dt^2) = 0,$$

and an approximating function consisting of a constant plus a ramp.

The optimum third order and fourth order filters are

$$T_3(s) = s^3 + (3\omega_o^2/5)s,$$

which has imaginary axis roots within the stopband range, and

$$T_4(s) = s^4 + (5\omega_o^2/7)s^2$$

which is similar but with a repeated zero at $s = 0$.

The repeated imaginary axis roots of $T_2(s)$ and $T_4(s)$ indicate instability of the approximating differential equation, but such instability of the observed "plant" is of no particular concern in observer design since observer eigenvalues may be placed arbitrarily [2-3, 18-19].

Properties of the Optimal Transfer Functions

It is particularly convenient at this point to consider T(s) in the factored form

$$T(s) = s(s+\alpha)(s^2+\beta_1 s+\gamma_1)(s^2+\beta_2 s+\gamma_2)\ldots,$$

where the real root term $(s+\alpha)$ is present if T(s) is of even order and is deleted if T(s) is of odd order. Then

$$|T(j\omega)|^2 = \omega^2(\omega^2+\alpha^2)(\omega^4+\beta_1^2\omega^2-2\gamma_1\omega^2+\gamma_1^2)(\omega^4+\beta_2^2\omega^2-2\gamma_2\omega^2+\gamma_2^2)\ldots$$

The performance measure is

$$J = \int_0^{\omega_o}|T(j\omega)|^2 d\omega$$

and one obtains

$$\partial J/\partial\alpha = \int_0^{\omega_o}\omega^2(2\alpha)(\omega^4+\beta_1^2\omega^2-2\gamma_1\omega^2+\gamma_1^2)\ \ldots\ d\omega$$

Setting this derivative to zero, one has $\alpha = 0$ in view of the nonegativity of the remaining factors in the integrand. Further,

$$\partial J/\partial\beta_1 = \int_0^{\omega_o}2(\omega^2+\alpha^2)(2\beta_1\omega^2)(\omega^4+\beta_2^2\omega^2-2\gamma_2\omega^2+\gamma_2^2)\ldots d\omega,$$

from which it is evident that $\beta_1 = 0$, and, similarly, that $\beta_2 = \beta_3 \ldots = 0$.

Taking the remaining partial derivatives such as $\partial J/\partial\gamma_1$ gives the further conditions for minimization of J, but in a difficult form. Having derived the important result that, depending upon the transfer function order, the optimal T(s) is either an even or an odd polynomial, we now return to the serial polynomial notation.

General Solution

For an even or odd polynomial,

$$T(s) = s^n + a_{n-2}s^{n-2} + a_{n-4}s^{n-4} + \ldots,$$

and

$$|T(j\omega)|^2 = [\omega^n - a_{n-2}\,\omega^{n-2} + a_{n-4}\omega^{n-4} - \ldots]^2$$

Interchanging the order of integration and differentiation in taking $\partial J/\partial a_i$, one has

$$\partial|T(j\omega)|^2/\partial a_i = 2\omega^i[\omega^n - a_{n-2}\omega^{n-2} + a_{n-4}\omega^{n-4} - \ldots]$$

so that

$$\partial J/\partial a_i = 2\int_0^{\omega_o}[\omega^{n+i} - a_{n-2}\omega^{n+i-2} + a_{n-4}\omega^{n+i-4} - \ldots]d\omega.$$

Equating to zero, there results the system of linear algebraic equations

$$\frac{\omega_o^{n+i-1}}{n+i-1}a_{n-2} - \frac{\omega_o^{n+i-3}}{n+i-3}a_{n-4} + \ldots = \frac{\omega_o^{n+i+1}}{n+i+1},$$

or

$$\frac{1}{n+i-1}(\omega_o^{-2}a_{n-2}) - \frac{1}{n+i-3}(\omega_o^{-4}a_{n-4}) + \ldots = \frac{1}{n+i+1},$$

$$i = (n-2),\ (n-4),\ \ldots$$

which may be solved to obtain $(\omega_o^{-2}a_{n-2})$, $(\omega_o^{-4}a_{n-4})$, ...

The next few optimal transfer functions that result are as follows:

$$T_5(s) = s^5 + (1.11)\omega_o^2 s^3 + (0.238)\omega_o^4$$

$$T_6(s) = s^6 + (1.27)\omega_o^2 s^4 + (0.353)\omega_o^4 s^2$$

$$T_7(s) = s^7 + (1.62)\omega_o^2 s^5 + (0.734)\omega_o^4 s^3 + (0.0816)\omega_o^6 s$$

$$T_8(s) = s^8 + (1.80)\omega_o^2 s^6 + (0.969)\omega_o^4 s^4 + (0.147)\omega_o^6 s^2$$

In the next section, it is shown how the above results can be used in two signal processing applications.

5. Examples

Observing a System State

Consider the system

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}-2 & 1\\ -3 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}1\\ 1\end{bmatrix}f(t)$$

$$y = \begin{bmatrix}1 & 1\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}$$

where f(t) is an unknown input signal. The transfer functions relating f(t) to the state signals $x_1(t)$ and $x_2(t)$ are

$$T_1(s) = (s+1)/(s^2+2s+3)$$
$$T_2(s) = (s-1)/(s^2+2s+3)$$

respectively.

Each of these transfer functions exhibits frequency response which approaches -20 decibels per decade above about 2 radians per second. If the amplitude of the spectrum of $f(t)$ is bounded at high frequencies, the effects upon the state of the high frequencies, in comparison with the lower frequencies in $f(t)$, will be small.

For example, if $f(t)$ is a square wave of radian frequency 1, the third harmonic amplitude in x_1 and x_2 will be approximately 0.15 as large as the fundamental. The relative amplitude of the fifth harmonic will be less than 0.05.

Taking $\omega_o = 10$ to be the effective band limit of the input signal, $f(t)$ is approximated by $f'(t)$, where f' is chosen to satisfy the optimal third order approximation

$$(d^3f'/dt^3) + (3\omega_o^2/5)(df'/dt) = 0,$$

the augmented system equations become

$$x' = \begin{bmatrix} -2 & 1 & 1 & 0 & 0 \\ -3 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -60 & 0 \end{bmatrix} \dot{x}'$$

$$y = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \end{bmatrix} x'.$$

An observer of this system will provide estimates of the system state and, if desired, of $f(t)$. Examples of analog computer simulations of such problems appear in [19].

__Observing a Filtered Signal__

In many practical applications, for example in biomedical instrumentation and economic modeling, it is desired to estimate inaccessible signals which may be considered to be unknown inputs to filters wherein only the output signals are available.

One approach is to consider the unknown and inaccessible signals to be generated, approximately, by free systems of the type (1). The available output signals are then considered to be produced by the larger system consisting of the actual system augmented by the approximating equations. An observer of the augmented system will then generate estimates of the inaccessible signals [18].

Consider an inaccessible signal $f(t)$ which is processed by a simple low-pass filter to produce the available signal $y(t)$ where

$$Y(s) = [1/(s+1)]F(s) = G(s)F(s).$$

This filter is represented by the system

$$\dot{x} = -x + f(t)$$

$$y = x.$$

Approximating $f(t)$ by $f'(t)$, where f' satisfies the optimal first order differential equation

$$\dot{f}' = 0,$$

gives the augmented system

$$\begin{bmatrix} \dot{x} \\ \dot{f}' \end{bmatrix} = \begin{bmatrix} -1 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ f' \end{bmatrix}$$

$$y = x(t).$$

An identity observer of the augmented system, with eigenvalues chosen to be -3 and -4 is

$$\begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \end{bmatrix} = \begin{bmatrix} -7 & 1 \\ -12 & 0 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} + \begin{bmatrix} 6 \\ 12 \end{bmatrix} y(t),$$

and the signal $z_2(t)$ observes $f'(t)$, and thus provides an estimate of $f(t)$.

The observer transfer function is

$$H(s) = \frac{12(s+1)}{s^2 + 7s + 12},$$

which is identical to that obtained by the conventional means in which $H(s)$ is chosen to be the inverse of $G(s)$, with additional poles added so that it has more poles than zeros [23]. It is particularly interesting that the methods described here show how conventional inverse filter techniques may be logically and easily extended to approximations of arbitrarily high order in multivariable systems.

In this connection, it is worth emphasizing that the optimal approximating functions are a constant for the first order and a constant plus ramp for the second order, functions which are quite familiar in classical circuit and control theory. The results for the third and higher orders, however, depend upon ω_o, the band limit of the signal $f(t)$.

6. Conclusion

It has been shown how inaccessible signals which are taken to be bandlimited, but otherwise arbitrary, can be suitably modeled for inclusion in a variety of signal processing applications. The model amounts to approximate representation of the signal as a solution of a differential equation with the coefficients of the latter determined to minimize the integral-square passband error of the approximation. Properties of these approximations were investigated and the coefficients of the representations up to order 8 were determined. Applications of the results were illustrated in two examples, the first yielding reconstruction of the state of a second-order dynamic system which is subject to a bandlimited disturbance signal, the second the recovery of such a signal from output measurements alone. In the latter case, an intimate connection with inverses was established.

Additional research has indicated that the approach given here can be extended to certain problems in digital signal processing [24].

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The problem of approximating a bandlimited but otherwise arbitrary signal by a free solution to a linear-time-invariant, differential equation is solved. Optimal solutions for the equation parameters are derived for equations of any dynamic order. Applications are discussed and examples in state reconstruction and inverse filtering in the presence of unknown disturbances are given.

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